Problem Chosen	2021	Team Control Number
D	MCM/ICM Summary Sheet	2108841

Know More about Music: Analysis on Network and Characteristics

Summary

Music is important human collective experience. Previous music influences new works and artists, and promotes the progress and development of music. The characteristics of the songs can explain the degree of influence. It can also reflect external factors that cause music change. We build influence networks and develop models to measure musical influence and identify the occurrence of revolutionary and evolutionary.

In Task 1, we create artists' influence directed networks, then we define five indicators, including NL centrality, two indicators of direct influence and other two from time dimension. After that, we utilize *Analytic Hierarchy Process* to build a model which measures the degree of influence. Lastly, we use our music influence model in 1960's artists and visualize the result.

In Task 2, we use *Principle Component Analysis* to screen out characteristics that mainly affect music similarity. Then we use principle component scores to calculate Euclidean distance, which is called music similarity degree. According to the value of music similarity degree, we utilize *K-means clustering algorithm* and find that artists are more similar whin genres.

In Task 3, we draw *box-plot* to find characteristics that distinguish genres. Based on *K-means clustering algorithm* results, we find the characteristics of genres vary greatly over time. Then we visualize the changes of artists' s numbers of each genres over time. After that, we create direct networks of genres to find and analyze the influence relationships between genres.

In Task4, to explore if 'influencers' actually being affected by the followers' s music, we build *Casual Bayesian network* and use *Causal Mutual Information* to measure the *Causal Power* between a pair of parent and child nodes. Then, we choose artists to analyze different characteristics' influence degree, indicating that different characteristics' influence is different.

In Task5, we draw lines charts of years' average music characteristics, and find some of them reflect musical evolutions caused by technology development and the overall changes of music characteristics. Then we define conditions revolutionaries must meet, *Intentional attack algorithm* is used to attack revolutionaries and find drop of *Network Efficiency* after attacking.

In Task6, we define dynamic influencer who has the largest music influence. After drawing linear graph, we find when dynamic influencer' characteristics change, are followed by characteristics changes throughout the genre . The bridge in the middle is other artists in the genre are affected by the changes of the characteristics of the dynamic influencer' songs.

In Task7, based on our work and the history of the development of American music, we analyze the effects and reactions of music and external environment.

Keyword: influence network, interaction effect, Casual Bayesian network, centricity clustering algorithm, Principle Component Analysis, Euclidean distance

Contents

1 Introduction

1.1Background

Music has played an important role in human's collective experience. And understanding how the previous music influence the new produced ones and musical artists enables us having a better knowledge of how music evolves with time passing by. Song characteristics may have similarities between the former artists and followers, such as lyrics, rhythm, structure and so on. The artists influencing the followers both of whom have similar style contributes to major shifts and changes in musical genre. What is more, external events like technological advances and major world events also make a great influence on the shifts on genre.

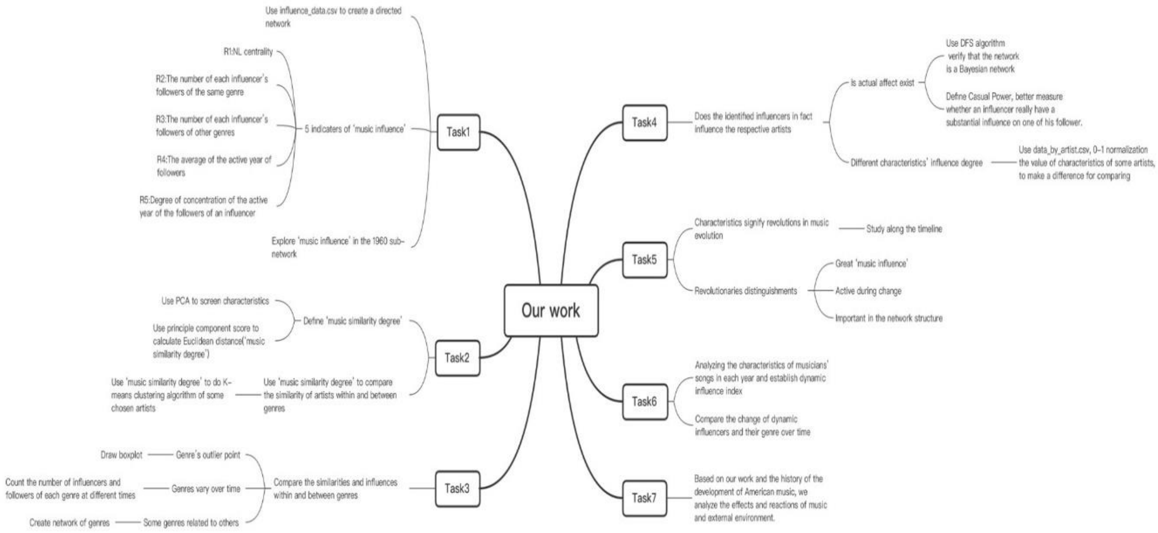
Through the consideration of songs and their musical characteristics, we are able to capture the interrelationship among musical artists, thus having a better understanding of how music evolves over time in society.

1.2 Restatement of problems

As the request of ICM, our team are provided with the following tasks:

- Create a(multiple) directed network(s) of musical influence (from followers to influencers) based on *influence_data.* Parameters capture *'music influence'* should be developed. Create a subnetwork of directed influencer network and make a description and evaluation.
- Measures of music similarity should be developed using *full_music_data* and/or the two summary data sets. Make sure whether artists are similar in same genres or between genres.
- Make comparisons of similarities and influences between and within genres. Think about the differences between genres, how genres change over time and the explore the relationship between and within genres.
- Use our measures of similarities to see whether identified influencers influence the respective artists indeed and the music created by their followers. Distinguish if some music characteristics more contagious or their influence are similar.
- Identify whether significant revolutions can be indicated by the data. And find the artists who represent revolutionaries in our network.
- Choose one genre and analyze its influence process of musical evolution. In addition, identify indicators revealing the dynamic influencers and explain the changes of genre(s) and artist(s) over time.
- The reflection of cultural influence of music in time or circumstances(Alternaltively, social, political or technological changes) through our work should be presented.
- One-page Summary Sheet should be presented to ICM, illustrating the efficiency of our approach to understanding the influence of music through networks and the adaptability of our solutions to more data.

1.3 Overview of our work



2 General Assumptions and Variable Description

2.1 Assumptions

- The artist influence network is an directed graph.
- Regardless of the influence of various special factors (such as personality charm, good appearance) on an artist's music influence, only from the artist's position and role as a node in the network structure.
- If one follower considers that a influencer had an impact on him, we define that there should be a directed edge between the follower and the influencer, from the influencer point to the follower.
- By default, the influence of artists without followers through the network is 0.

2.2 Variable Description

Table 1: Notations

Symbol	Definition
E	Edges of network
N	Nodes of network
W_{uv}	Value of major contribution of edge between u and v
$N(u)$	The sum of degree of nodes around u and the degree of u
S	music influence
$d(x,y)$	music similarity degree
CMI	Causal Mutual Information
CP	Causal Power
S	Sensitivity

3 Model Establishment and Solutions

3.1 Task1: Influence network construction and music influence parameter capture

3.1.1 Influence network construction

Considering the relationship between influencers and followers, the network of musical influence can be described as an *adjacency matrix*[1] in Data Structure. And adjacency matrix can be generated to perform network analysis and it reflects the connections between influencer nodes and follower nodes. A graph $G=(N,E)$ is defined by a set of nodes N and edges E. A weighted directed edge between node i and j is defined via $A_{ij}=w_{ij}$ **错误!未找到引用源。**. There are as many directed edges as pairs of followers and influencers. Adjacency matrix of the 5000 artists 'relationships is in **Figure1**. What is more, we use Gephi to visualize the relationships of the 5000 artists. The nodes in the network represent the influencers and followers and the number of nodes corresponds to the number of artists. The size of a node circle depends on
its degree of nodes, and the larger the degree of nodes, the larger the circle. In **Figure2**, the degree is the sum of out-degrees and in-degrees. Every link is one-way, from one influencer to one follower. However, because the data sample is too large, only part of the adjacency matrix is displayed, and the visualization network is too dense. In figure2, different colors do not have specific meaning.

	David Guetta	24hrs	…….	Verse Simmonds
Bob Schneider	0	0	……	0
Bob Seger	0	0	……	0
Bob Sinclar	1	0	…..	0
…..	…..	……	…..	……
T-Pain	0	1	…..	1

Figure1 adjacency matrix of 5000 artists

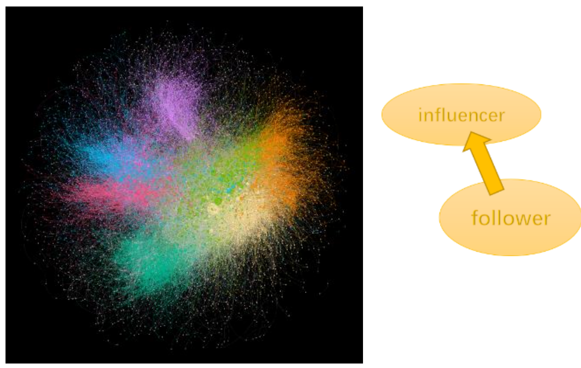


Figure2 visualization of the relationships of 5000 artists

3.1.2 Music influence parameter capture model

When measuring *music influence* of each artist, we start from five aspects. The indicators of each aspect and their descriptions are listed in **Table2**.

Then we calculate the index value of each aspect and then standardize it. Finally, *Analytic Hierarchy Process (AHP)* [3]method is used to get the total ability measurement index of the music influence of each artist.

Indicator	Description
R1	Use the NL centrality to evaluate the influence of nodes in the network
R2	Each influencer's number of followers of the same genre
R3	Each influencer's number of followers of other genres
R4	The average of the active year of the followers
R5	Degree of concentration of the active year of followers of an influencer

Table 2: Indicators and descriptions of music influence measurement

(The active year refers to the decade that the artist began their music career.)

R2 and R3 characterize the influence of influencers in his own genre and other genres. R4 and R5 represents the influence of influencer in the time dimension.

(1)**R1**: When calculating the influence of a node, the NL centrality algorithm takes the degree of the node and the weight of the edge connected to the node into account. Edge e_{uv} is between node u and node v. The importance of e_{uv} is defined as:

$$I_{e_{uv}} = \frac{U}{\lambda} \tag{1}$$

Where $U = (k_u - p - 1) \bullet (k_v - p - 1)$ measures the edge connectivity of e_{uv}, k_u and k_v represent the degree of node u and node v, p is the number of triangles formed by the e_{uv} side and other sides, λ is replacement index of e_{uv}, which is $\lambda = p/2 + 1$.

The nodes and edges in the network influence each other. It is not difficult to find that the more important the node, the more important the edge connected to the node. In the same way, the greater the contribution to the important degree of the edge, the greater influence the edge will make to the node. But the nodes at the two ends of each side have different attributes and different contribution to the important degree of the edge. So we define u's contribution to the important degree of e_{uv}. r_{uv} is major contribution ratio and W_{uv} is the value of major contribution.

$$r_{uv} = \frac{k_u - 1}{k_u + k_v - 2} \tag{2}$$

$$W_{uv} = I_{e_{uv}} \bullet r_{uv} \tag{3}$$

The definition of NL centrality algorithm is as follows:

$$R1 = \sum_{u \in \Gamma(v)} N(u) + \sum_{u \in \Gamma(v)} W_{uv}$$

(2) **R5**: We use the size of the sample standard deviation to measure the concentration of followers to each influencer

$$R5 = \left[\frac{1}{n-1} \sum_{n=1}^{n} (x_i - \bar{x})^2 \right]^{\frac{1}{2}} \tag{4}$$

Where n refers to the number of followers of the influencer. $\bar{x}$ is the average of active year of all the followers to an influencer. x_i is the active year of the ith follower to an influencer.

After making 0-1 normalization $x^{*}=\frac{x-x_{\min}}{x_{\max}-x_{\min}}$, we use the AHP to give the judgment matrix, and calculate the weight of each variable. Judgment matrix are shown as follows and the judgement results shows that it passes the consistency test.

$$\begin{array}{ccccc} R1 & R2 & R3 & R4 & R5 \end{array}$$
$$\begin{bmatrix} 1 & 5 & 7 & 8 & 9 \\ 1/5 & 1 & 3 & 5 & 7 \\ 1/7 & 1/3 & 1 & 4 & 6 \\ 1/8 & 1/5 & 1/4 & 1 & 2 \\ 1/9 & 1/7 & 1/6 & 1/2 & 1 \end{bmatrix} \tag{5}$$

Table 3：The judgment matrix weight vector calculation report

Consistency index	0.10196
Consistency ratio	0.091036
Consistency check result	Pass
Eigenvalue:	5.4078
Weight vectors	R1:0.58591 R2: 0.21346 R3:0.1214 R4:0.047557 R5:0.031673

In this way, we get the formula for calculating the index of music influence of each artists. I is the model to reflect artist's influence.

$$\text{S}=0.58591R1+0.21346R2+0.1214R3+0.047557R4+0.031673R5 \tag{6}$$

3.1.3 Select subnet and make analyses.

When talk about the subset of the network of music influence, we choose the year 1960 to draw a sub-network by Gephi. It is because through data analyzing, we find that the Beatles have the biggest music influence. Beatles's active start is 1960 (The decade that the influencing artist began their music career) and there are several artists who have a huge number of followers began their career in 1960.

We use Gephi to visualize the directed network. **In this network, the degree of nodes is redefined. The degree of each node is the value S of the artist.** The size of the node represents its degree, if the circle is big, the value S is big, vice versa. The link is one-way, from influencers to followers. And the different genres have been distinguished by colors.

When we set the number of constraints (The number of follower), we get a clear directed network, which is in **Figure3**:

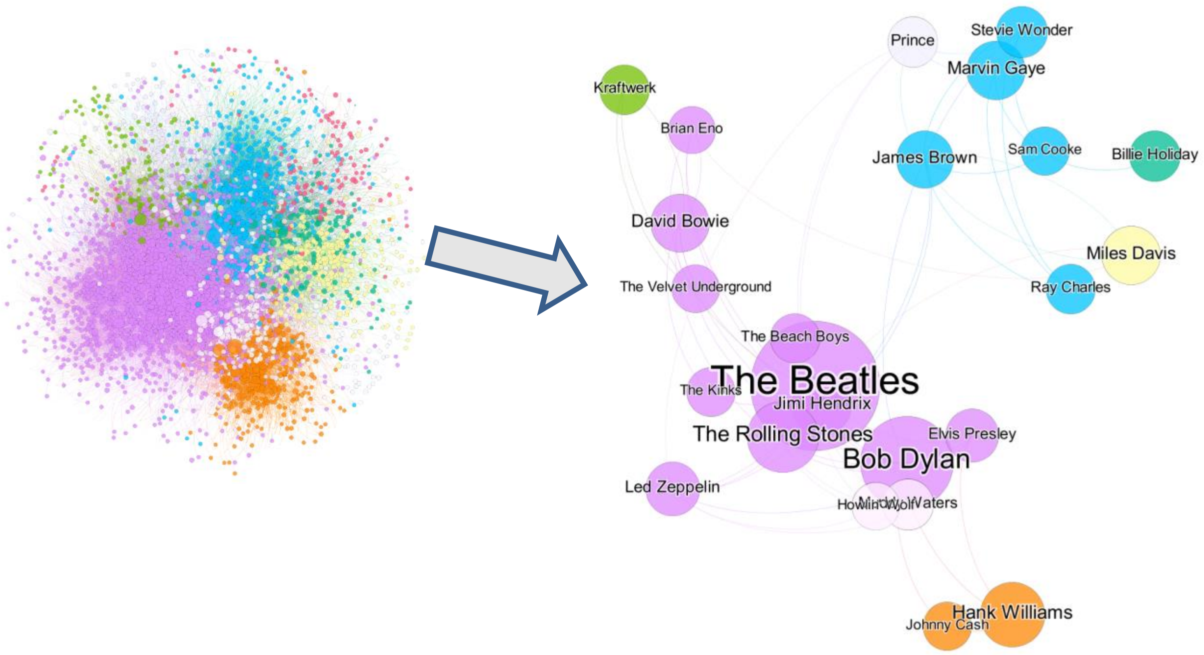


Figure3 Visualization of sub-network

3.2 Task2 Music similarity model ＆ Artists similarity analysis

3.2.1 Music similarity degree

We define **music similarity degree** to measure music similarity.

We use the following 15 variables to do **Principle Component Analysis (PCA)** to build a model to measure song similarity degree:

$$Y = C1 \bullet X1 + C2 \bullet X2 + C3 \bullet X3 + ... + C14 \bullet X14 + C15 \bullet X15 \tag{7}$$

Where C1~C15 refer to the 15 variables, X1~X15 are the weights of these 15 variables. The 15 variables are in table3. Y is a measurement of music similarity degree based on songs

Table4 15 variables of song characteristics

1	2	3	4	5
danceability	energy	valence	tempo	loudness
6	7	8	9	10
mode	key	acousticness	instrumentalness	liveness
11	12	13	14	15
speechiness	explicit	duration_ms	popularity	year

Firstly, we use 0-1 normalization to normalize these 15 variables. And we use the 0-1 normalization to do **Principal Component Analysis (PCA),** the results are as follows:

Through calculation, we get 5 principle component scores y1,y2,y3,y4,y5.

Table5 PCA results and results of song similarity calculation

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.713	24.757	24.757	3.713	24.757	24.757	2.973	19.823	19.823
2	1.646	10.971	35.727	1.646	10.971	35.727	1.844	12.291	32.114
3	1.336	8.904	44.631	1.336	8.904	44.631	1.730	11.531	43.645
4	1.200	8.000	52.631	1.200	8.000	52.631	1.309	8.727	52.372
5	1.119	7.459	60.090	1.119	7.459	60.090	1.158	7.719	60.090
6	.992	6.616	66.706						
7	.938	6.250	72.956						
8	.830	5.532	78.489						
9	.798	5.319	83.807						
10	.741	4.937	88.744						
11	.663	4.417	93.161						
12	.373	2.487	95.648						
13	.329	2.195	97.843						
14	.204	1.357	99.201						
15	.120	.799	100.000						

Extraction Method: Principal Component Analysis.

Component Matrix[a]

	Component				
	1	2	3	4	5
Zscore(danceability)	.358	-.639	-.247	.363	.007
Zscore(energy)	.845	.001	.243	-.165	.168
Zscore(valence)	.370	-.776	.073	-.005	.198
Zscore(tempo)	.275	-.001	.289	-.416	.180
Zscore(loudness)	.822	-.036	.095	-.150	.033
Zscore(mode)	-.046	-.140	.003	-.488	-.563
Zscore(key)	.044	.009	.036	.332	.492
Zscore(acousticness)	-.818	-.121	-.011	.134	-.149
Zscore(instrumentalness)	-.415	.277	-.001	.014	.289
Zscore(liveness)	.061	.130	.724	-.037	-.050
Zscore(speechiness)	.132	-.017	.597	.468	-.238
Zscore(explicit)	.274	.227	.129	.492	-.369
Zscore(duration_ms)	.014	.482	.006	.007	.337
Zscore(popularity)	.694	.303	-.371	.091	-.150
Zscore(year)	.747	.361	-.286	.094	-.145

Extraction Method: Principal Component Analysis.

a. 5 components extracted.

3.2.2 Measurements of similarity between the two subjects

We use Euclidean distance to measure the similarity between the two songs which is as follows:

$$d(x,y)=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2+(x_3-y_3)^2+(x_4-y_4)^2+(x_5-y_5)^2} \tag{8}$$

x_1, x_2, x_3, x_4, x_5 are 5 principle component scores of one song. y_1, y_2, y_3, y_4, y_5 are 5 principle component scores of another song. $d(x,y)$ is the measurement of musical similarity of the two songs, it is defined to be **music similarity degree**. If $d(x,y)$ is equal to 0, the two songs are exactly alike, the smaller $d(x,y)$ is, the more similar the two songs are, and vice versa.

In addition, when we consider music similarity degree at the artist level, we can use data in *data_by_artist* to do a principal component analysis, then calculate the Euclidean distance.

3.2.3 K-means clustering algorithm of artists between and within genres

We calculated Euclidean distance, and then conduct K-means clustering algorithm on artists to find out whether artists within genre more similar than artists between genres.

We selected 50 artists from Folk and 50 artists from Reggae, normalized data of *data_by_artist* was used for principal component analysis, and then calculated Euclidian distance to conduct K-means clustering algorithm. And results are in **Figure4.**

Number of Cases in each Cluster

Cluster	1	52.000
	2	48.000
Valid		100.000
Missing		.000

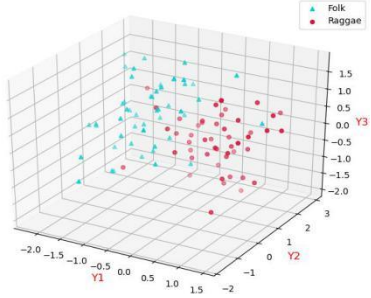


Figure4 Results of K-means

We notice that only 2 out of 50 artists are divided into another category. Therefore, it can be concluded that artists of the same genre have more similar characteristics than of different genres.

3.3 Task Similarities and influence in and within genres

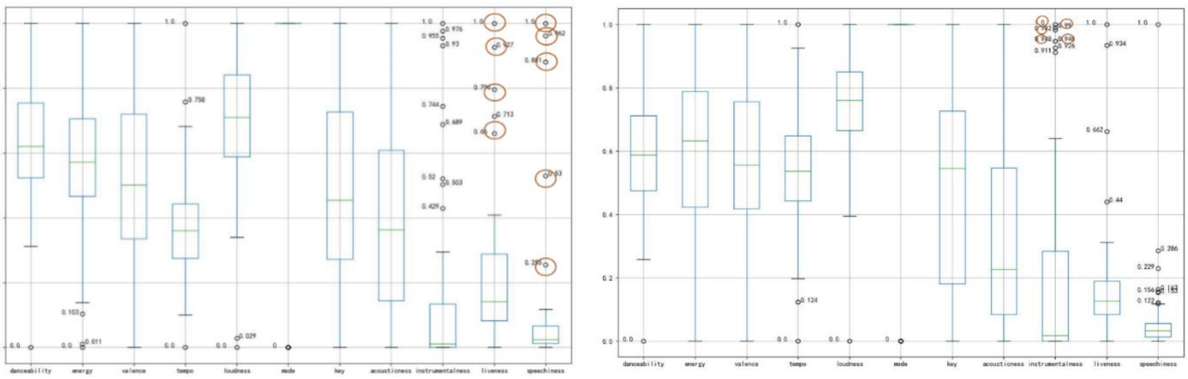
3.3.1 Genre's outlier point and feature values

The reason why genre is different is that they have characteristics that distinguish them from other genres. We draw boxplot, which described the discrete of distribution in a relatively stable way, to distinguish genre's specialness of characteristics and take Comedy/Spoken and jazz as examples. Choose 5 singers from the target genre and 60 singers from other genres.

In **Figure5**, we notice that the 5 artists' speechiness whose genre is Comedy/Spoken all belong to outliers, and 4 of liveness is outside the outliers. So, we are supposed to believe that speechiness and liveness are the discriminating and special characteristics of genre Comedy/Spoken. Similarly, we use this method to filter out the discriminating and special characteristic of jazz, which is instrumentalness.

Some obvious features of genres are well filtered out by drawing box-plot, and they are very vivid presented.

Figure5 the result after filtering outliers after drawing box plot



Comedy/Spoken Jazz

3.3.2 Genres vary over time

When considering the change of genre over time, we consider the change in the number of artists of the genre over time, with dimensions of follower and influencer

When we view the number of influencer and followers every 10 years, initial observations are found on time dimension. The trend of 20 genres' number of artists are visualized in **Figure6**. The horizontal coordinates time is decade that the artist began their music career. Lines of different colors on the line chart represent the trends of different genre. Since the number of pop/rocks is too large compared to the others, we draw its line chart separately.

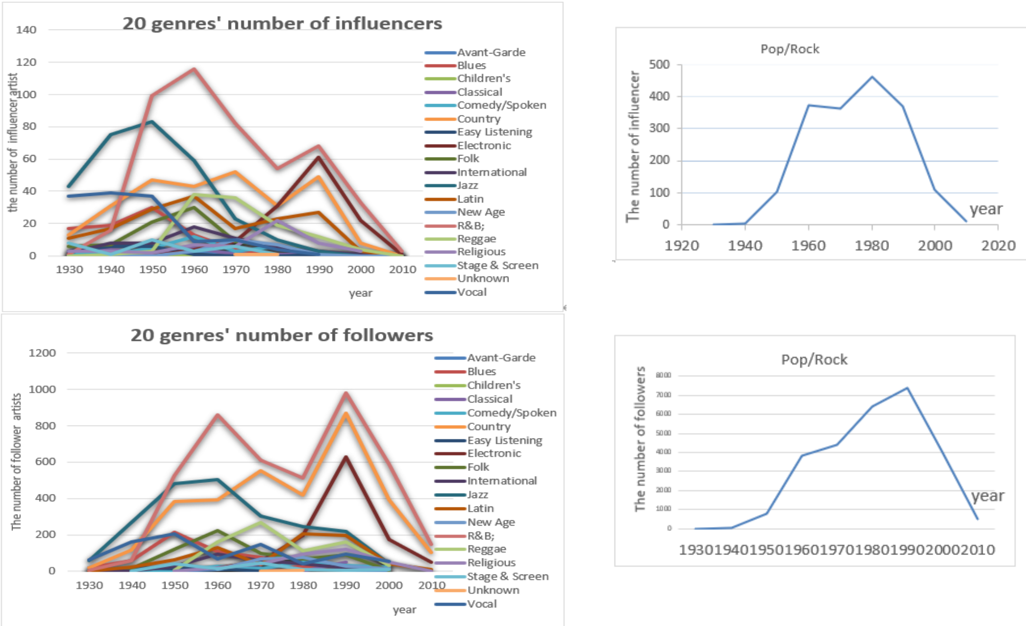


Figure6 the trend of the number of artists over time

(The first figure of pop/Rock is about the influencer and the second is the follower's)

(1) From the line chart, we can generally see that there are spikes with time passing by, which may be related to technological change and some social factors, and it will be discussed later.

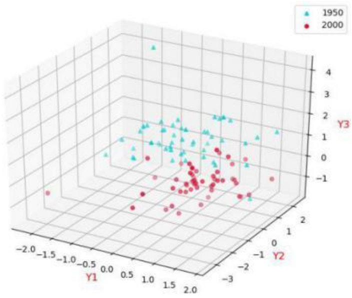
(2) It can be noted that for most genres, the period of time with most influencers and followers is between 1950 and 1990, while the number of influencers and followers is relatively small in 1930 and 2010.

(3) Whether from the overall line chart or the pop/rock line chart, we can see that the fluctuation of followers over time is consistent with the fluctuation trend of the number of influencers, but the fluctuation of followers lags behind influencers on time dimension.

(4) Take the Pop/Rock influencer chart as an example, Pop/Rock genre began to appear in about 1940 and developed rapidly from 1940 to 1960. With a turning point in about 1970 after a little decrease in 1960, followed by a slightly increase, and then steadily decreased for a period of time from 1990. However, there were very few influencers around 2010.

3.3.3 The characteristics of the one genres on time dimension

We still use *the K-means clustering method*, using the artists similarity degree which is one aspect of music similarity degree. 50 artists of Pop/Rock from 1950 and 2000 are selected 7for clustering and the results are in **Figure7**.



Number of Cases in each Cluster

Cluster	1	43.000
	2	57.000
Valid		100.000
Missing		.000

Figure7 Clustering results of songs on time dimension

When clustering based on average characteristics, most artists from 1950 and 2000 are separated, and belong to their own active year. So we concluded that the characteristics of genre itself have changed a lot over time.

3.3.4 The interconnectedness of genres

We find the influence relationships not only among the same genre, but also between genres, so we construct a directed network of genres' influence relationship, using the same way as mentioned in 3.1. However, in the network of genres, the size of the circle of nodes depends on the out-degree of this node, which means the number of followers influenced by the influencer. The link in the network points from the influencer's genre to the follower's genre. **Figure8** shows the visualization results. The size of the arrow head represents how the genre is affected by another. The bigger the size is, the bigger the influence is.

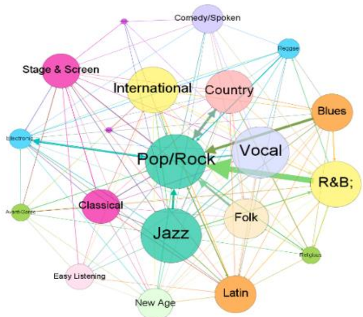


Figure8 visualization results of genre influence network

(1) As shown in **Figure8**, a genre not only influenced by other genres but also affects multiple genres, so there is direct and indirect influence between genres.

(2) Direct connection refers that from one node to another node only needs to pass through one edge, and indirect connection refers that the accessibility needs two or more edges. The relationship between the two genre is greater when they are direct connection, and when the two genres 'accessibility has to pass through another genre, the relationship between them is not very close.

(3) It can be seen from the network that Pop/Rock is related to many other genres, and genres like Vocal, Country, International also highly related to others. Take R＆B as an example, it affected Pop/Rock the most compared with other genres, and itself was affected by Jazz, blues.

3.4 Task4 The effectiveness of influencer and the influence differences of characteristics

3.4.1 Bayesian networks to determine the influence

We use DFS to traverse the entire artist network, and we get a directed acyclic network. The link in the network is the directed line segment pointed by the influencers to the followers, and each parent node and child node directly form the possible causal relationship, which is a typical *Causal Bayesian Network*. For each sub-network in the network, the causal model is formed between the parent nodes and the child nodes. We use Causal Mutual Information (CMI) to measure the Causal Power (CP) between parent and child nodes in the Causal model. For any causal model G, the causal mutual information of cause C and effect E is:

$$CMI(C,E)=\sum_{e\in E,c\in C} p(e|c)\log\frac{p(e|c)}{p(e)} \tag{9}$$

For specific causes $C=c$ and effects $E=e$, the causal intensity is

$$CP(c,e)=p(e|c)\log\frac{p(e|c)}{p(e)} \tag{10}$$

The higher the value of causality intensity is, the stronger the causal relationship between C and E is.

We randomly choose a sub-network to do analyses, and the Tank is the parent node, Rotimi is the child node. According to the data of *music influence*, we set the probability table of Tank. In the table, the column of YES represents the probability of exerting effects on other nodes, and the column of NO represents the probability of not exerting effects on other nodes. We use t_0 and t_1 to represent the two possible causes in the cause set T.

Table6 the probability table of Tank

Yes(t_0)	No(t_1)
0.25	0.75

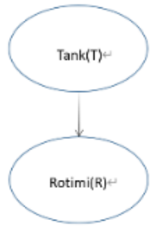


Figure9 pointing diagram

In the music influence data calculated by us, Tank's music influence value is 0.123515199, ranking 2801 among 3783 artists whose music influence index is not 0, and its influence is relatively low. So we set $p(\mathrm{t\_0})=0.25$, $p(\mathrm{t\_1})=0.75$.

According to the music similarity degree of Tank and Rotimi, we set t the probability table of Rotimi. The Grade A column indicating that the effect exerted by Tank has a great influence on Rotimi, the column Grade B(r_1) indicates small influence, Grade C(r_2) refers that no material impact was produced.We use r_0, r_1 and r_2 to represent the three possible effects in the effect set R.

Table7 the probability table of Rotimi

	Grade A(r_0)	Grade B(r_1)	Grade C(r_2)
t_0	0.285	0.536	0.179
t_1	0	0	1

We calculated that the music similarity degree of Tank and Rotimi was 2.963, while the average value of music similarity degree in the whole network was 3.317.

When $T=\mathrm{t\_0}$, we set that the music similarity degree between the parent node and the child node is 0, the probability of the effect exerted by the parent node to the child node with a large impact is 1, and the probability of having a small impact or no impact is 0. When music similarity is 3.317, the probability of the influence exerted by the parent node on the child node is 0.2, 0.6 and 0.2 respectively. We assume that in the process of music similarity degree increasing from 0 to 3.317 (the average value of the whole network), the probability of the influence of the three parent nodes on the child nodes varies uniformly and continuously. In this way we can the following between Tank and Rotimi:

$$\begin{cases} p(\mathrm{r\_0} \mid \mathrm{t\_0}) = 0.285 \\ p(\mathrm{r\_1} \mid \mathrm{t\_0}) = 0.536 \\ p(\mathrm{r\_2} \mid \mathrm{t\_0}) = 0.179 \end{cases}$$

When $T = \mathrm{t\_1}$, it is obviously that $p(\mathrm{r\_0} \mid \mathrm{t\_1}) = p(\mathrm{r\_1} \mid \mathrm{t\_1}) = 0, p(\mathrm{r\_2} \mid \mathrm{t\_1}) = 1$. Since Rotimi thinks Tank has an influence on him, we think Tank has an influence on Rotimi, namely $T = \mathrm{t\_0}$. We know that the chain rule for Bayesian networks is defined as $p(X) = \prod p_i(X_i \mid Par(X_i))$, among which $Par(X_i)$ is a collection of parent nodes. So we know that:

$$\begin{cases} p(\mathrm{r\_0}) = p(\mathrm{r\_0}, \mathrm{t\_0}) + p(\mathrm{r\_0}, \mathrm{t\_1}) = p(\mathrm{r\_0} \mid \mathrm{t\_0})p(\mathrm{t\_0}) + p(\mathrm{r\_0} \mid \mathrm{t\_1})p(\mathrm{t\_1}) \\ p(\mathrm{r\_1}) = p(\mathrm{r\_1}, \mathrm{t\_0}) + p(\mathrm{r\_1}, \mathrm{t\_1}) = p(\mathrm{r\_1} \mid \mathrm{t\_0})p(\mathrm{t\_0}) + p(\mathrm{r\_1} \mid \mathrm{t\_1})p(\mathrm{t\_1}) \\ p(\mathrm{r\_2}) = p(\mathrm{r\_2}, \mathrm{t\_0}) + p(\mathrm{r\_2}, \mathrm{t\_1}) = p(\mathrm{r\_2} \mid \mathrm{t\_0})p(\mathrm{t\_0}) + p(\mathrm{r\_2} \mid \mathrm{t\_1})p(\mathrm{t\_1}) \end{cases} \quad (11)$$

(It is the calculation formula of conditional probability)

The results of the calculation is: $\begin{cases} p(\mathrm{r\_0}) = 0.07125 \\ p(\mathrm{r\_1}) = 0.134 \\ p(\mathrm{r\_2}) = 0.79475 \end{cases}$

So CP is calculated as $\begin{cases} CP(\mathrm{t\_0}, \mathrm{r\_0}) = p(\mathrm{r\_0} \mid \mathrm{t\_0}) \log \dfrac{p(\mathrm{r\_0} \mid \mathrm{t\_0})}{p(\mathrm{r\_0})} = 0.3951 \\ CP(\mathrm{t\_0}, \mathrm{r\_1}) = p(\mathrm{r\_1} \mid \mathrm{t\_0}) \log \dfrac{p(\mathrm{r\_1} \mid \mathrm{t\_0})}{p(\mathrm{r\_1})} = 0.7431 \\ CP(\mathrm{t\_0}, \mathrm{r\_2}) = p(\mathrm{r\_2} \mid \mathrm{t\_0}) \log \dfrac{p(\mathrm{r\_2} \mid \mathrm{t\_0})}{p(\mathrm{r\_2})} = \text{-}0.2668 \end{cases}$

According to the calculated value of causality intensity, it is unlikely that Tank exerted effect on Rotimi and does not have substantial influence on Rotimi. It can be estimated that Tank did have substantial influence on Rotimi. And the probability of having a small effect is high, the probability of having a large effect is low.

According to our algorithm, when the music influence of the parent node itself is very small, if it is determined that it has an effect on the child nodes, then the probability that the effect has a substantial impact is very high. According to this, the result is very consistent with the fact. Because when an influencer is not influential enough, if one of his followers claims that he has been influenced by the influencer, we can know that the follower is unlikely to learn about the influencer under the influence of external factors such as social trends and cultural transmission. He is more likely to know and understand this influencer purely through the way of music, so followers' like to this influencer should be based on personal musical aesthetics, thus the music of an influencer has higher material influence on the music of a follower.

3.4.2 Different characteristics' influence degree

When talk about characteristics' influence degree to particular artist's music, we should make it clear whether some characteristics are more contagious or they are similar. We choose two artists Alice cooper whose genre is Pop/Rock and Shaggy whose genre is Reggae.

According to our observation of all characteristics, the following characteristics are selected for the study of influence degree to particular artist ’s music.

Table8 Selected characteristics

danceability	energy	valence
tempo	loudness	acousticness
instrumentalness	speechiness	duration_ms

We find out all influencers of Alice, and calculated the different value of these characteristics of all influencers and Alice. After normalization of the series of values, we calculate the average value of the difference value of each characteristic. Normalized and then sorted the average values of these difference value of each characteristic, we obtained the ranking of characteristics' influence on Alice. We do the same thing for shaggy which is from a different genre to test a little bit further, and the results are shown in **Table9**.

Table9 the sorting results of characteristics influence degree

	instrumentalness	energy	speechiness	duration_ms	loudness	danceability
Alice Cooper	1	0.980	0.807	0.802	0.073	0.63
	acousticness	valence	temple			
	0.596	0.0435	0			

	instrumentalness	acousticness	energy	speechiness	loudness	tempo
Shaggy	1	0.594	0.502	0.479	0.428	0.385
	valence	duration_ms	danceability			
	0.186	0.164	0			

As shown in the table, we draw the conclusion that different characteristics paly different roles in influencing the respective artists and some characteristics are more contagious.

3.5 Task5 Screening characteristics signify revolutions and revolutionaries

3.5.1 Characteristics signify revolutions in music evolution

By analyzing the changes of music characteristics over time (from 1921 to 2020), we found that some characteristics like tempo, valence, and liveness have not changed much over time. And characteristics like key has large fluctuations vary with the year. After comprehensive analysis, we selected the following variables that have greater fluctuations over time. And these selected characteristics signify music evolution to some extent. And major leaps of these characteristics might refer to revolutions in music evolution.

(1) *danceability, energy, loudness*

danceability measures how suitable a track is for dancing, and the larger the value, the more suitable it is for dancing. *Energy* is a measure representing a perception of intensity and activity, and larger the value is, more intense/energetic the music is. *Loudness* is the quality of a sound that is the primary psychological correlate of physical strength. Higher the value is, the music is more noisy.

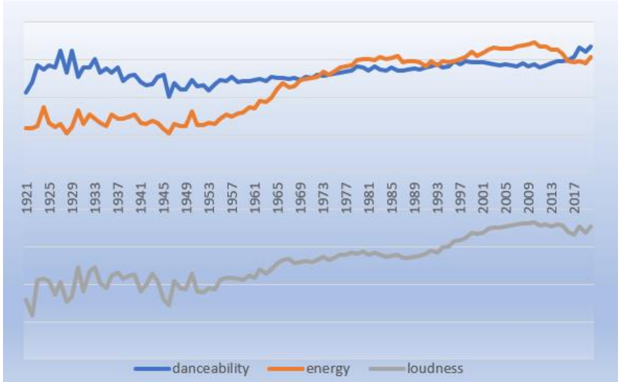


Figure10 three characteristics change over time

These three characteristics have similar changing trend over time. And they all represent the excitement and intensity of a song.. This coincides with the increasing popularity of pop/rock, electronic and other music at that time. And these three values have increased significantly from 1957 to 1977, which are exactly related to the popularity of disco and the maturity of rock music during that period.[4]. In addition, electrical engineers at RCA designed the first programmable electronic synthesizer, named the RCA Mark II Sound Synthesizer. The synthesizer opened up the possibilities of electronic music, which can be indicated in **Figure10**,and **Figure11** is drawn with given data.

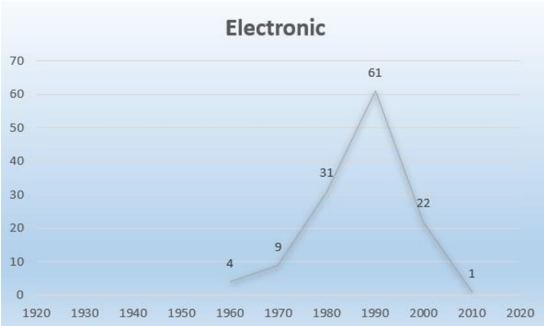


Figure11 the number of artists belong to electronic change over time

In conclusion, we think these three characteristics can reflect people's increasing love of passionate music and the increasing popularity of passionate music and the change of music genres' relationship with technological progress and innovation

(2) *popularity*

popularity is the characteristic that reflects how frequently songs are being played now.

As can be seen from the **Figure12**, people's preference for songs that appeared between 1956 and 1971 has increased considerably. People' love for songs that came out after 1971 increased more modestly when recent releases are not taken into account. This means that from 1951 to 1971, song styles changed a lot, and after 1971, they changed relatively little

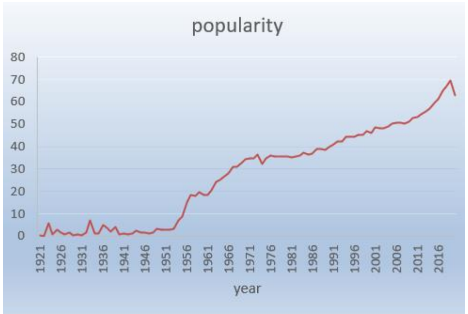


Figure12 popularity changes over time

We can draw a conclusion that the characteristic *popularity*'s leap signifies revolutions in musical evolution caused by social and policy factors, etc.

(3) *acousticness*

acousticness is a confidence measure of whether the track is acoustic without technology We can speculate that with more technology enhancements or electrical amplification being involved, the impact of sound technology will be greater to determine whether the track is acoustic.So this characteristic is related to the degree of impact of technological development on music.

In **Figure13**, there is a huge decline of *acousticness* from 1960s to 1980s, indicating that acoustic track is more dependent on technological factors. This period of time is exactly the time when electronic synthesizer and CD are put into use and the came out of first digital mixer.

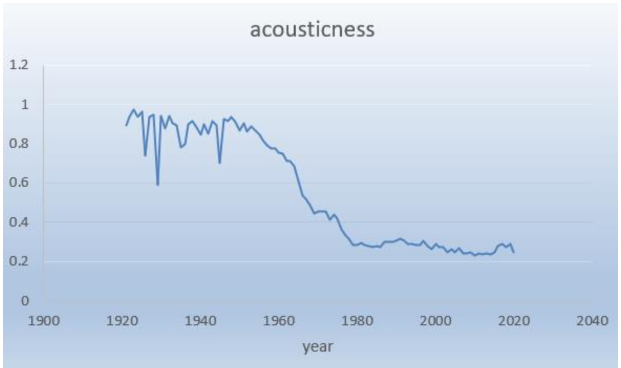


Figure13 acousticness changes over time

3.5.2 Revolutionaries distinguishments and Intentional attack algorithm

We consider the revolutionaries need to satisfy the following three factors:

(1) They have great music influence.

(2) They are active during a period of significant change in their genre.

(3) They are important nodes in a network of artists. And as revolutionaries, they are in the transitional position of the musical characteristics of their own genre when the genre changing from one style to another. If they are removed from the nodes, the structure of the network will change dramatically.

We focus on the Pop/Rock genre and sift out revolutionaries based on the following three factors：

(1) According to the music influence data, we screened out Pop/Rock artists ranked in the top 100 of all artists, with a total number of 67 musicians.

(2) Important periods of change in the Pop/Rock genre are:

1950-1960: The rise of rock and roll in the US.

1960-1970: Rock became vibrant, resulting in a large number of British rock bands occupying the local music market in the United States.

1970-1980: Heavy metal took off and the punk movement fired its first shots.

1980-1990: Punk gradually evolved into the new wave and post-punk style.

(3)We use *Intentional attack algorithm (IAA)* to attack the top 6 nodes and the other 4 random nodes in each category according to the chronological classification to observe the changes of the network.

Table9 the nodes which are attacked

1950	1960	1970	1980
Elvis Presley	The Beatles	Sex Pistols	Metallica
Chuck Berry	Bob Dylan	The Clash	Nirvana
Buddy Holly	The Rolling Stones	Brian Eno	R.E.M.
Little Richard	David Bowie	Ramones	The Smiths
Leonard Cohen	Led Zeppelin	Roxy Music	Madonna
Frank Zappa	Jimi Hendrix	Black Flag	Pixies
Ace Frehley	The Cyrkle	ZZ TOP	New Order
Bobby Darin	The Raincoats	Bad Finger	Annie Lennox
The Band	Steve Howe	Kansas	Peter Gabriel
The Rascals	madness	Love	ABBA

We use *Network Efficiency (NE)* to describe the overall state of the network：

$$NE = N*(N-1)/\sum_{i=1}^{N}\sum_{j=1}^{i}(1/d_{ij}) \tag{12}$$

N: The number of nodes in the network
d_{ij}: The value of the shortest connected path from node *i* to node *j*

Floyd algorithm is used to calculate the shortest connectivity distance between nodes. After *IAA* is used to attack nodes, the change of *NE* is used to reflect the overall change of the network. Collect *NE* after each attack to get a line graph.

According to the chart analysis, combined with the development process of rock history, the overall network efficiency showed a downward trend over time, which is closely related to the introduction of new blood in rock music in the early 1960s and the development of heavy metal in the 1970s. When the revolutionary node is deleted, the slope of the line graph is often significantly higher than that obtained by deleting other ordinary nodes, which reflects that the revolutionary node is more important to the network than other nodes. The changes of the network confirm the correctness of the three factors of selecting revolutionaries.

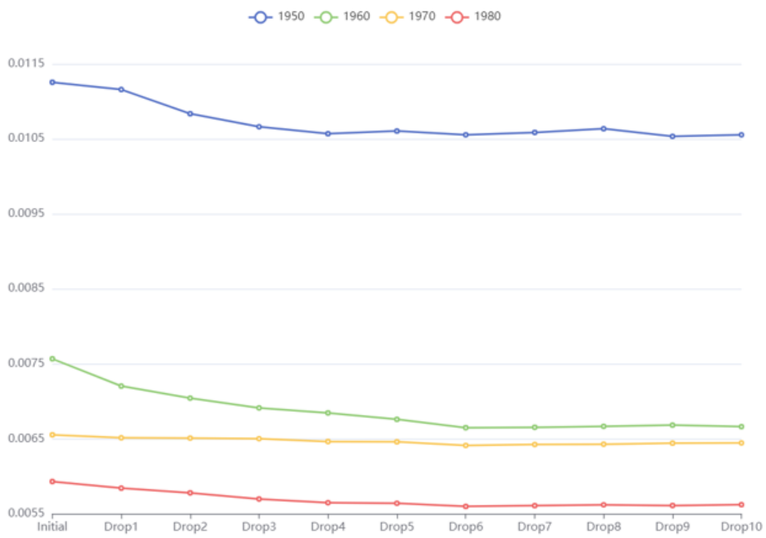


Figure14 Network Efficiency changes over time

3.6 Dynamic influencers´ influence on genres and artists

The music influence model *I* build in the first question can reflects the influence degree of influencers. We supposed that the dynamic influencers have great influence within the genre and the value of I can signify the degree of influence.

We illustrate dynamic influence as follows:

We consider that dynamic influencers refer to those who have great influence on genres and whose musical style changes dynamically over time in their musical career. Meanwhile, their own dynamic change will affects the whole genre, and other musicians will be affected too. Other musicians 'changing of music style will leads to the dynamic change of genres.

Since other musicians in this genre(Pop/Rock) will absorb and internalize the dynamic influencer's song, thus having more similar characteristics, We suspect that the dynamic changes of the entire genre are similar to the influencer change, but with a slight lag. We select all the songs of Beatles from 1963 to 1970, who is the most influential musician, and calculate the normalized average value of each song's characteristics, which is shown in **Figure15**. Then we select all the songs of other musicians in this genre except the Beatles from 1963 to 1970, and calculated the average value of each song's characteristics, which is shown in **Figure16**.

From the figure, we can see that our speculations of about how the genre(s) or artist(s) changed over time with the influence of dynamic influencer are right.

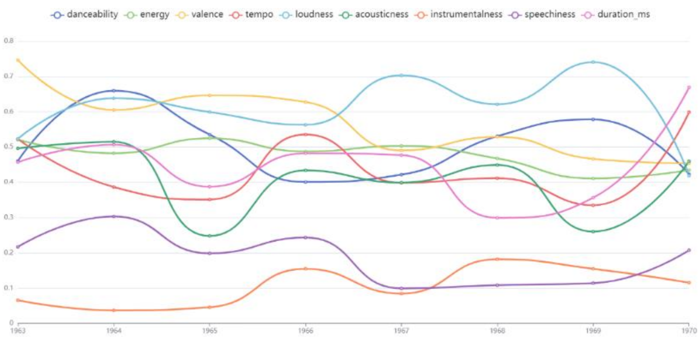


Figure15 the change of characteristics of dynamic influencer's songs over time

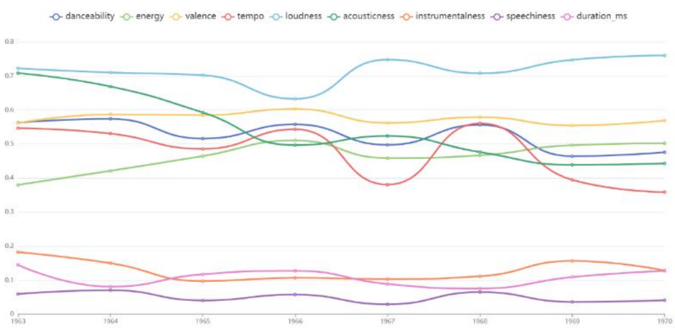


Figure16 the change of characteristics of the whole genre's songs over time

3.7 Musical influence and being affected by other factors

3.7.1 cultural influence of music in time and circumstances

(1) Music promotes cultural integration

We build the network of artist influence in 3.1 and genre influence in 3.3. We find that the mutual influence between songs and genres is very complicated. This influence is not single, but compound, and it is transitive throughout the network. And it is the mutual influence between all kind of songs, and the mutual reference and development between genres, which promoted the formation and development of American music. These transitive relationships also give rise to new genres. For example, the earliest rock came from blues, it has a strong taste of blues, and its own characteristics slowly began to form later.

In the time dimension, we can see the continuous blending of different genres produce new genres. In addition, this kind of fusion, promotes the communication between different races and cultures. For example, rap was mainly popular among the blacks at first, and then became popular all over the world, showing that music promotes the connection between different race groups.

(2) As a part of spiritual culture, music has obvious expressive, stimulating and inspiring power. It also has a more prominent power to transform beauty into truth and guides goodness from beauty. The music style and characteristics of each country are closely related to its social background and culture.

Rocky Mountain Way is written by John Denver in 1972, and at the end of the song, John expresses his concern that the Rocky Mountains will be harmed by the uncontrolled development of human beings, and deeply expresses his concern and love for nature and homeland. It has a popularity of 62, which is very high compared with other songs at that period of time. This shows that songs reflect on social phenomena and have a positive impact to social events. The same goes for *This Land is Your Land*. Woody Guthrie wrote this song to awaken the consciousness of democracy. This song has become a symbol of American spirit and has been covered by many singers in the later ages. The influence of these time-tested songs has been passed down from generation to generation and has a profound impact on culture.

3.7.2 Social political and technology changes' reflection in our work

(1) Social and political changes

In our work, as shown in **Figure10**, the rise of professional music in the United States was mainly between 1910-1950, which is the between two world wars (1918-1945).

From the First World War to the early 1930s, American industry, science and technology played an important role in the world. The financial and economic advantages provide important material conditions for the stability of American domestic life and the development of culture and art. With the economic prosperity, American public opinion began to call for professional music of the nation. At the same time, American music and music education were receiving more and more attention.[6].

We draw the conclusion that that the prosperity of music is related to social and political stability.

(2) Technology changes

We have discussed in 3.5.1 that characteristics like *danceability, energy, loudness* and *acousticness* can reveal the influence of technological development on music. We also mentioned that the increase of the number of electronic artists is related to the invention of the synthesizer.

When talk about acousticness, we discussed that inventions like CD and digital mixer all have effects on the characteristics of music, especially electronic music. In addition, due to technological progress, the generation of musical instruments is more scientific and the sound will also change. All of this can be reflected by the changes of music's characteristics.

All in all, changes in these three factors may affect changes in music. These changes are mainly reflected in the leaks and inflection points of changes in music-related characteristics.

4 Sensitivity analysis

When studying the Evaluation model in **TASK 4**, we use causal mutual information to measure the causal power between a pair of parent-child nodes in a causal Bayesian network. Through the calculation of conditional probabilities, the corresponding causal strengths of various causal relationships between a pair of parent-child nodes are obtained, and then we judge the likelihood of various causal relationships by the causal power which is measured. In this model we set $p(\mathrm{t\_0}) = 0.25$ and $p(\mathrm{t\_1}) = 0.75$ .Now we keep the probabilities set based on the music similarity degree between the parent and child nodes, which are:

$$\begin{cases} p(\mathrm{r\_0} \mid \mathrm{t\_0}) = 0.285 \\ p(\mathrm{r\_1} \mid \mathrm{t\_0}) = 0.536 \\ p(\mathrm{r\_2} \mid \mathrm{t\_0}) = 0.179 \end{cases}$$

Then we reduce 10 times by 10% each time, to analyze the change of CP 's sensitivity to $p(\mathrm{t\_0})$ as $p(\mathrm{t\_0})$ decrease. Sensitivity is calculated as

$$S(CP, p(\mathrm{t\_0})) = \frac{dCP}{dp(\mathrm{t\_0})} \frac{p(\mathrm{t\_0})}{CP}$$

It means that at one point (a,b), if $p(\mathrm{t\_0})$ reduce by 1%, CP will reduce by $S(CP, p(\mathrm{t\_0}))\%$ (if $S(CP, p(\mathrm{t\_0}))\% < 0$, then CP will increase by $-S(CP, p(\mathrm{t\_0}))\%$)

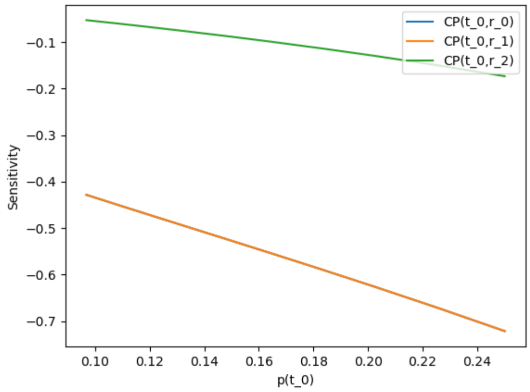


Figure17 the function images of $S(CP)$ - $p(\mathrm{t\_0})$

The result of sensitivity calculation are:

$$\begin{cases} S(CP(\mathrm{t\_0}, \mathrm{r\_0}), p(\mathrm{t\_0})) = -\dfrac{1}{\log \dfrac{1}{p(\mathrm{t\_0})}} \\ S(CP(\mathrm{t\_0}, \mathrm{r\_1}), p(\mathrm{t\_0})) = -\dfrac{1}{\log \dfrac{1}{p(\mathrm{t\_0})}} \\ S(CP(\mathrm{t\_0}, \mathrm{r\_2}), p(\mathrm{t\_0})) = \dfrac{0.147 p(\mathrm{t\_0})(5.587 - 4.587 p(\mathrm{t\_0}))}{(1 - 0.821 p(\mathrm{t\_0}))^2 \log(\dfrac{0.179}{1 - 0.821 p(\mathrm{t\_0})})} \end{cases}$$

The values of $S(CP)$ at $p(\mathrm{t\_0}) = 0.250$ and $p(\mathrm{t\_0}) = 0.097$ are showed in table:

Table10 The values of $S(CP)$ at $p(\mathrm{t\_0}) = 0.250$ and $p(\mathrm{t\_0}) = 0.097$

p(t_0)	0.250	0.097
S(CP(t_0,r_0),p(t_0)	-0.721	-0.428
S(CP(t_0,r_1),p(t_0)	-0.721	-0.428
S(CP(t_0,r_2),p(t_0)	-0.173	-0.053

From the **Figure17** and **Table10** we consider that the sensitivity of $CP(\mathrm{t\_0,r\_0})$ and $CP(\mathrm{t\_0,r\_1})$ to $p(\mathrm{t\_0})$ are well, and $CP(\mathrm{t\_0,r\_2})$ is not sensitive to $p(\mathrm{t\_0})$.

5 Evaluation of the model

5.1 Strengths

- The *music influence* model use *NL* centrality as primary factor and attach four secondary factors, get the appropriate weights of each factor through *AHP*, and complete a more comprehensive assessment of music influence.
- The *music similarity* model defines Euclidean distance calculated from the principle component score from PCA as music similarity degree, and using it to do k-means clustering , which has a good consistency.
- The *Casual Power* model treat the network as *causal Bayesian network*, and analyze the problem in combination with causal model.
- The network exhibits good robustness while using *IAA* attack nodes.

5.2 Weaknesses

- Our Model do not have enough analysis of songs and do not find a good way to determine the genre of the song. Because songs in *full_music_data.csv* do not have a corresponding genre tag, and due to the diversity of the singer's creative style, the genre of a song's creator cannot infer the genre of the song. We do not come up with a good way.
- The *Casual Power* model do not provide program of the causality calculation in the entire network, it has opportunity to be improved.

6 Document to the ICM Society

1.The value of using our approach to understanding the influence of music through networks

①We set up the index of music influence to measure the influence of different artists on music and build an artist influence network.

②We establish music similarity degree to measure the similarity among musicians, which further enriches the artists influence network.

③We build networks of artists by genre classification, extending the influence between musicians to a larger scale of influence between genres. Different genres serves as different nodes, the directed edge point from one genreto another genre which is influenced by it.

④We divide the process of music's influence into two parts based on causality:

(i) Whether the influence exerted by the influencer is passed on to the follower;

(ii)Whether the effect exerted exactly makes an influence on the follower affect;

By the study of influences based on Casual Power, we can better measure whether an influencer really have a substantial influence on one of his follower.

Also music has been refined into various characteristics, and we find some degree of similarity in the characteristics.

⑤We screened the major influencers in the music network. Through IAA and observing the change of network efficiency, our definition of major influencer is proved to be correct.

⑥ The dynamic influence index is established by analyzing the characteristics of musicians' songs in each year. We consider the dynamic influencer must has big music influence in his genre and their change over time can lead to changes in other artists, and these artists influence other artists in this genre and finally the entire genre will present a dynamic change, which is similar to the change of dynamic influencer but shows a lag in time.

⑦We find that the relationship between songs and genres is compound, songs and genres can often be linked by race or culture. Each country's musical style and characteristics are closely related to its social background and culture. The prosperity of music is related to social and political stability. These are such complex factors that make up the network of musical influences.

2. About how the work or solution changes with the enrichment of data:

For the explored data types, the data enrichment only increase the number of nodes and edges in the network, and the model and algorithm we use to explore the network properties are still applicable. With richer data types, the music network will produce more parts for us to mine and explore, leading to a deeper understanding of the impact of music through the network. For example, each song is accompanied by the nationality and race of the creator, as well as the precise positioning of the song in the genre, we can use these information to dig out the cultural background and era background of song creation. If every song is accompanied by lyrics, we can split the lyrics, or use the word vector principle to analyze the feelings expressed by the lyrics, or dig out the cultural connotation expressed by the creators through keyword analysis.

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[6] 黄颖 《浅谈美国音乐发展特点及其影响》湖南省长沙市明德中学

Appendix

(1) data

Principal component analysis results of some of artists

artist_name	Y1	Y2	Y3	Y4	Y5
Frank Sinatra	-2.88009052	1.163564282	5.291907642	-5.248721801	-6.554378767
Vladimir Horowitz	-5.596488286	-0.075574744	4.061584527	-4.390552093	-6.185957813
Johnny Cash	-1.058593452	2.548402853	4.453412416	-3.279700591	-4.206391045
Billie Holiday	-2.138798602	2.374988351	4.431967239	-3.763972515	-5.091768369
Bob Dylan	-1.296505138	1.029221678	4.487706159	-3.59928372	-4.850666861
The Rolling Stones	-0.774638512	0.919722774	4.113927219	-4.040449707	-6.030285065

Some of the data used in drawing box-plot:

influencer_name	er_ma	rtist_i	anceability	energy	valence	tempo	loudness	od	key	acousticness	strumentalne	liveness	speechiness	duration_ms
Pat Martino	Jazz	792583	0.490845345	0.093314383	0.269376024	0.556573504	0.39386296	0	0.636363636	0.99977346	1	0.058045908	0.052089396	0.4145286
Bill Frisell	Jazz	74397	0	0	0.007030712	0.123609154	0	0	0.727272727	0.994676314	0.990026247	0	0.015015714	0.583938405
Lennie Tristano	Jazz	242966	0.507149144	0.194956177	0.623392424	0.261730602	0.450453329	1	0	1	0.982093904	0.085232811	0.029139021	0.261155957
Medeski, Martin & Wood	Jazz	406583	0.585638229	0.606953479	0.73664521	0.794953992	0.727906345	1	0	0.483687387	0.947506562	0.170442255	0.055552322	0.317242074
Charlie Ventura	Jazz	101628	0.631736138	0.475209096	0.8533289	0.680875932	0.894514999	1	1	0.983462592	0.947506562	0.147578416	0.022989175	0.111659799

uencer_n	cer_main	rtist_i	nceabili	energy	valence	tempo	loudness	mode	key	ousticne	rumental	liveness	eechines	ration_m
Dick Greg	Comedy/Sp	211197	0.612903	0.594959	0.593867	0.284579	0.544078	1	0.090909	0.971944	2.94E-05	0.660169	1	0.395713
Brian Reg	Comedy/Sp	615937	0.635246	0.477579	0.275166	0.117576	0.407418	1	0.545455	0.844227	3.46E-05	1	0.962175	0.21653
Richard I	Comedy/Sp	295415	0.511223	0.726776	0.186294	0.100121	0.554404	1	0.545455	0.798649	0.000161	0.926567	0.880681	0.212564
Steve Mar	Comedy/Sp	43444	0.631592	0.619903	0.384583	0.448131	0.450222	1	0.909091	0.598631	0.00014	0.79591	0.529663	0.192691
Cheech &	Comedy/Sp	106298	0.705645	0.606892	0.537425	0.364523	0.726876	1	0.818182	0.430854	2.13E-05	0.288741	0.255268	0.199245

(2) code

AHP，matlab

```
clc
clear all
A=[1 5 7 8 9;1/5 1 3 5 7;1/7 1/3 1 4 6;1/8 1/5 1/4 1 2;1/9 1/7 1/6 1/2 1];
[n,n]=size(A);[v,d]=eig(A);r=d(1,1);CI=(r-n)/(n-1);
RI=[0 0 0.58 0.90 1.12 1.24 1.32 1.41 1.45 1.49 1.52 1.54 1.56 1.58 1.59];CR=CI/RI(n);
if  CR<0.10
    CR_Result=&apos;pass&apos;;
   else
    CR_Result=&apos;fail&apos;;
end
w=v(:,1)/sum(v(:,1));
w=w&apos;;
```

```
disp(&apos;The judgment matrix weight vector calculation report:&apos;);
disp([&apos;consistency index:&apos; num2str(CI)]);
disp([&apos;consistency ratio:&apos; num2str(CR)]);
disp([&apos;consistency check result:&apos; CR_Result]);
disp([&apos;eigenvalue:&apos; num2str(r)]);
disp([&apos;Weight vector:&apos; num2str(w)]);
```

Sensitivity analysis，python

```
import matplotlib.pyplot as plt
import sympy as sp
x0=[];y1=[];y2=[];y3=[]
pt0r0=0.285;pt0r1=0.536;pt0r2=0.179;pt1r0=0;pt1r1=0;pt1r2=1;pt0=0.25
x=sp.Symbol(&apos;x&apos;)
f0=pt0r0*sp.log(pt0r0/(pt0r0*x+pt1r0*(1-x)))
f1=pt0r1*sp.log(pt0r1/(pt0r1*x+pt1r1*(1-x)))
f2=pt0r2*sp.log(pt0r2/(pt0r2*x+pt1r2*(1-x)))
f0_=sp.diff(f0,x)*x/f0;f1_=sp.diff(f1,x)*x/f1;f2_=sp.diff(f2,x)*x/f2
print(f0_,f1_,f2_)
tempf0_=[];tempf1_=[];tempf2_=[]
for i in range(0,10):
    pt0=0.25*(9/10)**i
      x0.append(pt0)
for j in range(0,10):
      z0=float(f0_.evalf(subs={x:x0[j]}));z1=float(f1_.evalf(subs={x:x0[j]}));z2=float(f2_.evalf(subs={x:x
0[j]}))
      tempf0_.append(z0);tempf1_.append(z1);tempf2_.append(z2)
print(tempf0_[0],tempf1_[0],tempf2_[0])
print(tempf0_[9],tempf1_[9],tempf2_[9])
print(x0[9])
plt.plot(x0,tempf0_,x0,tempf1_,x0,tempf2_)
plt.xlabel(&apos;p(t_0)&apos;);
plt.ylabel(&apos;Sensitivity&apos;)
plt.legend([&apos;CP(t_0,r_0)&apos;, &apos;CP(t_0,r_1)&apos;, &apos;CP(t_0,r_2)&apos;], loc=&apos;
upper right&apos;)
plt.show()
```